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$n=1$ $\quad \sin^2\alpha = \frac{1}{2}\left(1 - \frac{\sin\alpha\cos 2\alpha}{\sin\alpha}\right)$

$\sin^2\alpha = \frac{1}{2}\left(1-(1-2\sin^2\alpha)\right)$ ✓

$n=k$ $\quad \sum_{i=1}^{k} \sin^2(i\alpha) = \frac{1}{2}\left(k - \frac{\sin(k\alpha)\cos(k+1)\alpha}{\sin\alpha}\right)$

$n=k+1$ $\quad \sum_{i=1}^{k+1} \sin^2(i\alpha) \overset{?}{=} \frac{1}{2}\left(k+1 - \frac{\sin(k+1)\alpha\cos(k+2)\alpha}{\sin\alpha}\right)$

$$\underbrace{\sum_{i=1}^{k}\sin^2(i\alpha)} + \sin^2(k+1)\alpha \overset{?}{=}$$

$$\frac{1}{2}\left(k - \frac{\sin(k\alpha)\cos(k+1)\alpha}{\sin\alpha}\right) + \sin^2(k+1)\alpha \overset{?}{=}$$

$$\frac{1}{2}k - \frac{1}{2}\cdot\frac{\sin(k\alpha)\cos(k+1)\alpha}{\sin\alpha} + 1 - \cos^2(k+1)\alpha \overset{?}{=}$$

$$\frac{1}{2}k+1 - \cos(k+1)\alpha\left[\frac{\sin k\alpha}{2\sin\alpha} + \cos(k+1)\alpha\right] \overset{?}{=}$$

$$\frac{1}{2}k+1 - \cos(k+1)\alpha\left[\frac{\sin(k\alpha) + 2\sin\alpha\cos(k+1)\alpha}{2\sin\alpha}\right] \overset{?}{=}$$

$$\frac{1}{2}k+1 - \cos(k+1)\alpha\left[\frac{\cancel{\sin(k\alpha)} + \sin(k+2)\alpha - \cancel{\sin(k\alpha)}}{2\sin\alpha}\right] \overset{?}{=}$$

$$\frac{1}{2}k+1 - \frac{\cos(k+1)\alpha\sin(k+2)\alpha}{2\sin\alpha}$$

$$\frac{1}{2}k + \frac{1}{2} + \frac{1}{2} - \frac{\cos(k+1)\alpha\sin(k+2)\alpha}{2\sin\alpha}$$

$$\frac{1}{2}k + \frac{1}{2} + \frac{\sin\alpha - \cos(k+1)\alpha\sin(k+2)\alpha}{2\sin\alpha}$$

$$\frac{1}{2}k + \frac{1}{2} + \frac{\cancel{\sin\alpha} - \frac{1}{2}\sin(2k+3)\alpha - \cancel{\frac{1}{2}\sin\alpha}}{2\sin\alpha}$$

$$\frac{1}{2}k + \frac{1}{2} - \frac{1}{2}\left(\frac{\sin(2k+3)\alpha}{2\sin\alpha} - \frac{\sin\alpha}{2\sin\alpha}\right)$$

$$\frac{1}{2}k + \frac{1}{2} - \frac{1}{2\cdot 2\sin\alpha}\left(\sin(2k+3)\alpha - \sin\alpha\right)$$

$$\frac{1}{2}k + \frac{1}{2} - \frac{1}{\cancel{2}\cdot 2\sin\alpha}\cdot\cancel{2}\sin(k+1)\alpha\cos(k+2)\alpha$$

$$\frac{1}{2}k + \frac{1}{2} - \frac{\sin(k+1)\alpha\cos(k+2)\alpha}{2\sin\alpha} =$$