12

$n=1$ $$\frac{1}{\sin 2\alpha} = \cot\alpha - \cot 2\alpha$$

$$= \frac{\cos\alpha}{\sin\alpha} - \frac{\cos 2\alpha}{\sin 2\alpha} = \frac{\cos\alpha}{\sin\alpha} - \frac{\cos 2\alpha}{2\sin\alpha\cos\alpha}$$

$$= \frac{2\cos^2\alpha - (2\cos^2\alpha - 1)}{2\sin\alpha\cos\alpha} = \frac{1}{\sin 2\alpha}$$

$n=k$ $$\sum_{i=1}^{k} \frac{1}{\sin 2^i\alpha} = \cot\alpha - \cot 2^k\alpha$$

$n=k+1$ $$\sum_{i=1}^{k+1} \frac{1}{\sin 2^i\alpha} \overset{?}{=} \cot\alpha - \cot 2^{k+1}\alpha$$

$$\underbrace{\sum_{i=1}^{k} \frac{1}{\sin 2^i\alpha}} + \frac{1}{\sin 2^{k+1}\alpha} \overset{?}{=} \cot\alpha - \cot 2^{k+1}\alpha$$

$$\cot\alpha - \cot 2^k\alpha + \frac{1}{\sin 2^{k+1}\alpha} \overset{?}{=}$$

$$\cot\alpha - \frac{\cos 2^k\alpha}{\sin 2^k\alpha} + \frac{1}{\sin 2^{k+1}\alpha} =$$

$$\cot\alpha - \frac{\cos 2^k\alpha}{\sin 2^k\alpha} + \frac{1}{\sin(2\cdot 2^k\alpha)} =$$

סי' זו' כפולה

$$\cot\alpha - \frac{\cos 2^k\alpha}{\sin 2^k\alpha} + \frac{1}{2\sin(2^k\alpha)\cos(2^k\alpha)} =$$

$$\cot\alpha + \frac{-2\cos(2^k\alpha)\cos(2^k\alpha) + 1}{2\sin(2^k\alpha)\cos(2^k\alpha)} =$$

cos של זווית כפולה

$$\cot\alpha - \frac{\cos(2\cdot 2^k\alpha)}{\sin(2\cdot 2^k\alpha)}$$

$$\cot\alpha - \frac{\cos(2^{k+1}\alpha)}{\sin(2^{k+1}\alpha)}$$

$$\cot\alpha - \cot(2^{k+1}\alpha) =$$