4. $n=1$ $\quad \left(1-\frac{1}{2}\right)=\frac{1}{2}$

$n=k$ $\quad \left(1-\frac{1}{k+1}\right)\left(1-\frac{1}{k+2}\right)\cdots\left(1-\frac{1}{2k}\right)=\frac{1}{2}$

$n=k+1$ $\quad \underbrace{\left(1-\frac{1}{k+2}\right)\left(1-\frac{1}{k+3}\right)\cdots\left(1-\frac{1}{2k}\right)}\left(1-\frac{1}{2k+1}\right)\left(1-\frac{1}{2k+2}\right)\overset{?}{=}\frac{1}{2}$

$$\frac{\frac{1}{2}}{1-\frac{1}{k+1}}\cdot\left(1-\frac{1}{2k+1}\right)\left(1-\frac{1}{2k+2}\right)\overset{?}{=}$$

$$\frac{\frac{1}{2}}{\frac{k+1-1}{k+1}}\cdot\frac{2k+1-1}{2k+1}\cdot\frac{2k+2-1}{2k+2}=$$

$$\frac{k+1}{2k}\cdot\frac{2k}{2k+1}\cdot\frac{2k+1}{2k+2}$$

$$\frac{k+1}{2k+2}=\frac{1}{2}$$