7
(356)

$n=1$ $\qquad 1\cdot 1+2\cdot 3 = \frac{1}{4}\left[(4-1)\,3^{2}+1\right]$ ✓

$n=k$ $\qquad 1\cdot 1+2\cdot 3+\dots+2k\,3^{2k-1} = \frac{1}{4}\left[(4k-1)\,3^{2k}+1\right]$

$n=k+1$

$$\underbrace{1\cdot 1+2\cdot 3+\dots+2k\,3^{2k-1}}+(2k+1)\,3^{2k}+(2k+2)\,3^{2k+1} \overset{?}{=} \frac{1}{4}\left[(4k+3)\,3^{2k+2}+1\right]$$

$$\frac{1}{4}\left[(4k-1)\,3^{2k}+1\right]+3^{2k}(2k+1+6k+6) \overset{?}{=}$$

$$\frac{1}{4}+\frac{1}{4}(4k-1)\,3^{2k}+3^{2k}(8k+7) \overset{?}{=}$$

$$\frac{1}{4}+\frac{3^{2k}}{4}(4k-1+32k+28) \overset{?}{=}$$

$$\frac{1}{4}+\frac{1}{4}\cdot 3^{2k}(36k+27) \overset{?}{=}$$

$$\frac{1}{4}+\frac{1}{4}\cdot 3^{2k}\left(9(4k+3)\right) \overset{?}{=}$$

$$\frac{1}{4}+\frac{1}{4}\cdot 3^{2k}\cdot 3^{2}(4k+3) \overset{?}{=}$$

$$\frac{1}{4}\left[1+3^{2k+2}(4k+3)\right] =$$