

15
556)

$n=1$

$$1 \cdot 2 + 2 \cdot 3 = \frac{4}{3} \cdot 1 \cdot 2 \cdot 3 \quad \checkmark$$

$n=k$

$$1 \cdot 2 + 2 \cdot 3 + \dots + 2k(2k+1) = \frac{4}{3} k(k+1)(2k+1)$$

$$n=k+1 \quad \underbrace{1 \cdot 2 + 2 \cdot 3 + \dots + 2k(2k+1)}_{\frac{4}{3} k(k+1)(2k+1)} + (2k+1)(2k+2) + (2k+2)(2k+3) \stackrel{?}{=} \frac{4}{3} (k+1)(k+2)(2k+3)$$

$$\frac{4}{3} k(k+1)(2k+1) + (2k+1)2(k+1) + 2(k+1)(2k+3) \stackrel{?}{=}$$

$$2(k+1) \left[\frac{2k}{3}(2k+1) + 2k+1 + 2k+3 \right] \stackrel{?}{=}$$

$$2(k+1) \left[\frac{4k^2 + 2k + 6k + 3 + 6k + 6}{3} \right] \stackrel{?}{=}$$

$$\frac{2(k+1)}{3} [4k^2 + 14k + 12] \stackrel{?}{=}$$

$$\frac{2(k+1)}{3} \cdot 2(2k^2 + 7k + 6) \stackrel{?}{=}$$

$$\frac{4(k+1)}{3} (k+2)(2k+3) \stackrel{?}{=}$$